A Non-Monetary Mechanism for Optimal Rate Control Through Efficient Delay Allocation

Tao Zhao[1] Korok Ray[2] I-Hong Hou[1]

[1]Department of ECE
Texas A&M University

[2]Mays School of Business
Texas A&M University

WiOpt'17

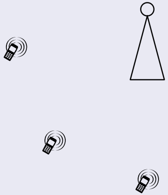
Motivation

Optimal Rate Control

- A wireless network with multiple clients
- Individual utility: function of request arrival rate
- Problem: Find optimal rates that maximize total utility

Game theory is needed.

- Clients: selfish and strategic
- Individual utility: private



Motivation

Existing Work

- Auction: e.g. VCG auction
- Direct payment between client and server

Issues of Monetary Mechanisms

- Monetary exchange requires addtional infrastructure.
- Pricing every packet? Impractical.

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Non-monetary mechanism!

How Non-Monetary?

Observation

- Each client suffers disutility based on experienced delay.
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Our Approach

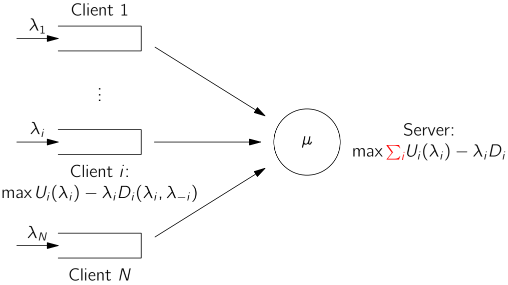
Use delay as the currency!

Main Contribution

A non-monetary mechanism by efficient delay allocation

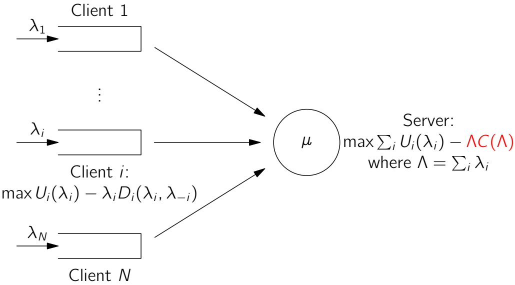
System Model

- One server: Average request service rate μ
- Client $i = 1, 2, \ldots, N$:
  - Average request arrival rate λ_i: adjustable
  - Utility $U_i(\lambda_i)$: increasing, twice differentiable, concave
  - Average request delay $D_i(\lambda_i, \lambda_{-i})$

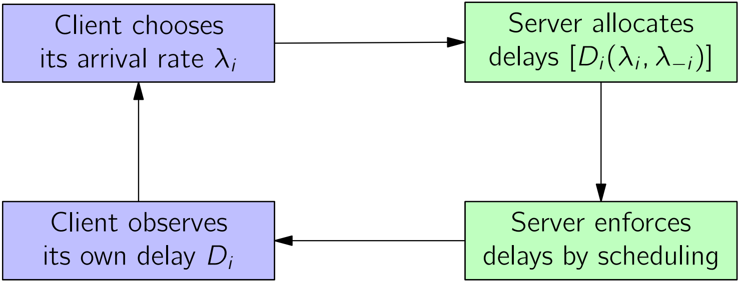


System Model

- Total average delay
  - Function of total average request arrival rate, $\Lambda := \sum_i \lambda_i$
  - Increasing and convex
  - Fitted by a $(N-2)$-order polynomial $C(\Lambda)$
- Assume feasible $\boldsymbol{\lambda} := [\lambda_i]$ satisfies $\Lambda < (1-\epsilon)\mu, \lambda_i > \lambda_\delta > 0$



Game Between Clients and Server



Nash Equilibrium and Efficiency

Definition

A vector $\tilde{\boldsymbol{\lambda}} := [\tilde{\lambda}_i]$ is said to be a Nash Equilibrium if $\tilde{\lambda}_i = \mathrm{argmax}_{\lambda_i} U_i(\lambda_i) - \lambda_i D_i(\lambda_i, \tilde{\lambda}_{-i}), \forall i$.

Definition

A rule of allocating delays, $[D_i(\cdot)]$, is said to be efficient if the vector that maximizes the total net utility, $\boldsymbol{\lambda}^* := [\lambda_i^*]$, is the only Nash Equilibrium.

Remark

Server's problem is to find and enforce the rule that allocates delays, $[D_i(\cdot)]$, to induce optimal choices of $[\lambda_i]$.

Non-Monetary Mechanism for Optimal Rate Control

1. Efficient Delay Allocation Rule
2. Scheduling Policy to Enforce Allocated Delays
3. Distributed Rate Control Protocol

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Property of Efficient Delay Allocation Rule

Server

$\boldsymbol{\lambda}^*$ is the solution to

$$\max \sum_i U_i(\lambda_i) - \Lambda C(\Lambda).$$

Hence,

$$U_i'(\lambda_i^*) = \frac{\partial}{\partial \lambda_i} \Lambda^* C(\Lambda^*)$$

Client

$\boldsymbol{\lambda}^*$ is the solution to

$$\max U_i(\lambda_i) - \lambda_i D_i(\lambda_i, \lambda_{-i}^*).$$

Hence,

$$U_i'(\lambda_i^*) = \frac{\partial}{\partial \lambda_i} \lambda_i^* D_i(\lambda_i^*, \lambda_{-i}^*)$$

Observation

Want $\Lambda C(\Lambda) - \lambda_i D_i(\lambda_i, \lambda_{-i}) =: R_i(\lambda_{-i})$, the external disutility, independent of λ_i

Delay Allocation Rule

Delay Allocation Rule

- $\lambda_i D_i(\lambda_i, \lambda_{-i}) = \Lambda C(\Lambda) - R_i(\lambda_{-i})$
- $R_i(\lambda_{-i}) = \sum_{j=1}^{N-1} \beta_i^j$
- $\beta_i^j = c_j \sum_{\boldsymbol{p} \in P_i^j} \frac{N-1}{N-G(\boldsymbol{p})} \frac{j!}{p_1! \cdots p_N!} \lambda_1^{p_1} \cdots \lambda_N^{p_N}$
- c_j: j-th order coefficient of polynomial $\Lambda C(\Lambda)$
- $P_i^j := \{\boldsymbol{p} = [p_n] \mid p_n \in \mathbb{Z}^*, \sum_{i=1}^{N} p_n = j, p_i = 0\}$
- $G(\boldsymbol{p})$ be the number of nonzero coordinates of $\boldsymbol{p}$

Theorem

Our rule of delay allocation $[D_i(\cdot)]$ is efficient.

An Example of Delay Allocation Rule

Example ($N = 3$)

β_i^j	$j = 1$	$j = 2$
$i = 1$	$c_1(\lambda_2 + \lambda_3)$	$c_2(\lambda_3^2 + 4\lambda_2\lambda_3 + \lambda_2^2)$
$i = 2$	$c_1(\lambda_1 + \lambda_3)$	$c_2(\lambda_3^2 + 4\lambda_1\lambda_3 + \lambda_1^2)$
$i = 3$	$c_1(\lambda_2 + \lambda_1)$	$c_2(\lambda_1^2 + 4\lambda_2\lambda_1 + \lambda_2^2)$

- External disutility R_i (row sum) is independent of λ_i
- Allocated disutility $\lambda_i D_i = \Lambda C(\Lambda) - R_i$
- Total disutility $\sum_i \lambda_i D_i = 3\Lambda C(\Lambda) - \sum_i R_i = \Lambda C(\Lambda)$

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Scheduling Policy

Problem

How to enforce target delay $D_i(\lambda_i, \lambda_{-i})$ for client i?

MRQ Scheduling Policy

Let $Q_i(t)$ be the queue length of client i at time t, and $g_i := \lambda_i D_i$. At time t, the MRQ policy schedules the client with the maximum relative queue length, defined as $Q_i(t)/g_i$.

Intuition

Eventually all relative queue lengths are equal on average in steady state, or equivalently, average queue length (delay) = target queue length (delay).

State Space Collapse

Theorem (State Space Collapse)

The efficient delay allocation rule is enforced by the MRQ scheduling policy in the heavy traffic regime.

Remark

- Heavy traffic: $\Lambda \to \mu$
- Show the deviation of the limiting queue length vector from the target queue length vector approaches 0
- Lyapunov drift based technique

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How Distributed?

We already know

- Our delay allocation rule is efficient.
- Our MRQ scheduling policy enforces the delay allocation rule.

Problem

How are the clients supposed to update their request rates distributedly to converge to the Nash Equilibrium?

Idea

- Projected gradient method: Centralized
- How to make it distributed?

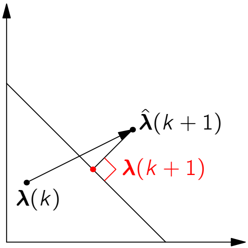
Centralized $\rightarrow$ Distributed

- Centralized update:

$$\hat{\boldsymbol{\lambda}}(k+1) = \boldsymbol{\lambda}(k) + \frac{\kappa(k)}{\eta(k)} \nabla \left[\sum U_i(\lambda_i) - \Lambda C(\Lambda) \right],$$

$$\boldsymbol{\lambda}(k+1) = P(\hat{\boldsymbol{\lambda}}(k+1))$$

- $\kappa(k)$: step size at the k-th iteration
- $\eta(k)$: Euclidean norm of the gradient
- P: projection to the feasible region s.t. $\lambda_i > \lambda_\delta$ and $\Lambda < (1-\epsilon)\mu$



Centralized → Distributed

- Distributed update:

$$\hat{\lambda}_i(k+1) = \lambda_i(k) + \frac{\kappa(k)}{\eta(k)} \left[U_i'(\lambda_i(k)) - \frac{\mathrm{d}[\Lambda C(\Lambda)]}{\mathrm{d}\Lambda} \right],$$

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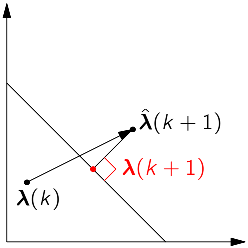

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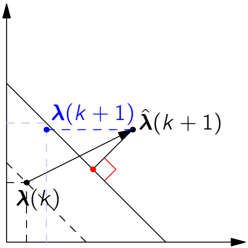

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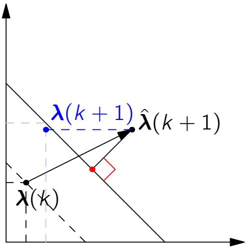
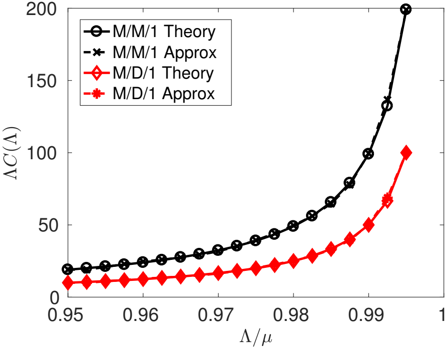
- $\kappa(k)$: step size at the k-th iteration
- $\eta(k)$: Euclidean norm of the gradient
- $\Lambda(k), \kappa(k), \eta(k)$, and $\frac{\mathrm{d}[\Lambda C(\Lambda)]}{\mathrm{d}\Lambda}$ are the same for all clients: Broadcast!



Simulations

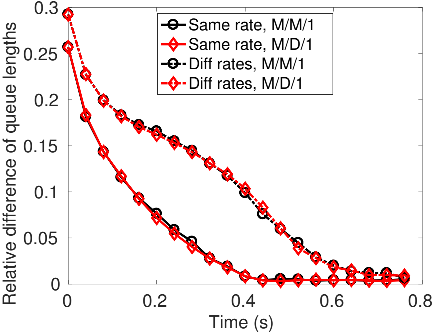
- Validate our non-monetary mechanism
  - Polynomial approximation assumption
  - State space collapse in scheduling
  - Optimality of distributed rate control protocol
- Baseline mechanism
  - FIFO (first-in-first-out) scheduling policy
  - Centralized projected gradient method for rate control
- Two systems: M/M/1 v.s. M/D/1
  - $N = 10$ clients
  - Poisson arrivals: $\Lambda = 0.99 \times 10^3\,\mathrm{s}^{-1}$
  - Exponential/Deterministic service time: $\mu = 1 \times 10^3\,\mathrm{s}^{-1}$

Polynomial Approximation



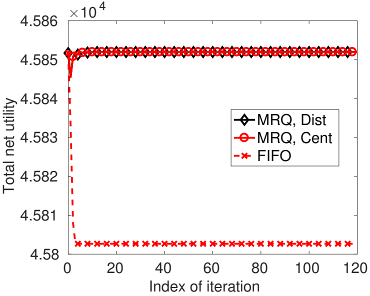
Total disutility $\Lambda C(\Lambda)$ v.s. Normalized total request rate Λ/μ

State Space Collapse

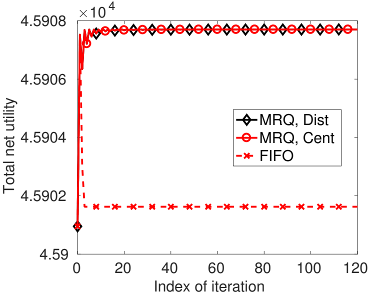


Normalized difference of relative queue lengths v.s. Time

Nash Equilibrium



M/M/1 system



M/D/1 system

Summary

Non-Monetary Mechanism for Optimal Rate Control

- Efficient delay allocation rule
- MRQ scheduling policy
- Distributed rate control protocol

Summary

Non-Monetary Mechanism for Optimal Rate Control

$$\text{Delay} = \text{Currency}$$
$$\text{Time} = \text{Money}$$

Thank you!

Tao Zhao
alick@tamu.edu

TEXAS A&M UNIVERSITY